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THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS.

About ten years ago an organization known as the National Council of Teachers of English was formed. Today it has a membership of 5,000. Under the leadership of the English Journal those 5,000 teachers are practically a unit in any project leading to the improvement of the teaching of English in the high schools of the United States. What a force for progress! Any question or problem confronting teachers of English is promptly handled by their National Council. Through their journal, all teachers of that subject are immediately aware of any significant experiment, study or change in curriculum in any part of the country.

During the same period high school mathematics courses have been assailed on every hand. So-called educational reformers have tinkered with the courses, and they, not knowing the subject and its values, in many cases have thrown out mathematics altogether or made it entirely elective. The individual teachers and local organizations have made a fine defense to be sure, but there could be no concerted action. Finally, the American Mathematical Association of America came to the rescue and appointed a committee to study the situation and to make recommendations. Already two valuable reports have been issued and others are in preparation. The pity of it is that this work, wholly in the realm of the secondary schools, should have to be done by an organization of college teachers. True they have generously called in high school teachers to help, but the fact is that it remained for the college people to initiate the work. They could do it because they possessed a live, vigorous organization.

To help remedy the existing situation the National Council

of Teachers of Mathematics was organized last spring at the N. E. A. meeting held in Cleveland.

A journal being absolutely necessary for the life and action of the Council, the Executive Committee immediately began negotiations for the control of The Mathematics Teacher, a magazine published quarterly by the Association of Teachers of Mathematics in the Middle States, and Maryland. These negotiations resulted in the National Council receiving The Mathematics Teacher free of debt, with the good will of the eastern association. It was a magnanimous action prompted solely by the highest motives and desires for the good of mathematics teachers in general. As President of the National Council, I publicly acknowledge the obligation and responsibility laid upon its officers and sponsors. That action is a challenge to us for unselfish devotion to the interests of mathematics in the secondary schools of the country. We are ready to accept the responsibility. In behalf of the Executive Committee and the editors of the journal, I pledge the best service that we are able to render. We do not claim superior ability in teaching or leadership. We do have, however, a consuming desire to give mathematics and the teaching of mathematics their proper place in the educational world. We desire to establish a medium through which the teachers of mathematics may express themselves on any question that pertains to their subject and its teaching. We realize the temporary character of our stewardship and, therefore, seek to establish the Council and its official journal upon a firm basis, before the leadership passes into other hands.

We also recognize the fact that unless we give value received for the membership fee no one will become a member a second time. We, therefore, intend to make good. We do not mean to explain. We also recognize the great inspiration and responsibility that a large membership will give us. Thus, the individual teachers have a duty as well as the officials of the Council. If the movement should fail the individual teacher who has not become a member and contributed his bit toward success will be as much to blame for failure as any one else.

Now, fellow teachers, what considerations should influence

us in deciding whether or not to join the Council? First, on the one hand, there is the fee of $2.00. That amount would buy forty issues of the *Saturday Evening Post*, or one seat at the opera or standing room at one football game. Second, things are all right as they are. There are enough associations and meetings already. A national organization cannot render any service not given by existing organizations.

On the other hand there are some good reasons for the National Council. First, it will at all times keep the values and interests of mathematics before the educational world. Instead of continual criticism at educational meetings, we intend to present constructive programs, by the friends of mathematics. We prefer that curriculum studies and reforms and adjustments come from the teachers of mathematics rather than from the educational reformers.

Second, it will furnish a medium through which teachers in one part of the country may know what is going on in every other part of the country. Significant reports and studies and experiments will be given wide circulation through the official journal. Otherwise they would be known only locally.

Third, the Council through its journal will furnish a medium of expression for all of the teachers of the country. Thus, a feeling of solidarity will be aroused. All teachers of mathematics will know that they are members of one family, working together under a common leadership.

Fourth, the Council will help the progressive teacher to be more progressive. It will also arouse the conservative teacher from his satisfaction and cause him to take a few steps ahead.

Fifth, the splendid work of the National Committee on Mathematical Requirements will be conserved and extended as time and new conditions may show the need.

Sixth, the Council should receive the support of every teacher of secondary mathematics because it intends to maintain a journal whose editors and writers are real class room teachers. In fact, the teachers themselves are the owners of the journal, if they will but join the Council.

The Committee is daily receiving assurances of co-operation and support from individuals and organizations in different parts of the country. Already the Council has been endorsed

by the Chicago Women's Mathematics Club, the Chicago Men's Mathematics Club, the Mathematics Section of the Central Association of Science and Mathematics Teachers, the Mathematics Teachers of the City of Chicago, the Association of Teachers of Mathematics in the Middle States and Maryland, the Mathematics Section of the Minnesota State Teachers' Association, the Mathematics Section of the Teachers' Conference held at the University of Illinois, The Mathematics Club of St. Louis, the Mathematics Section of the Missouri State Teachers' Association, the Mathematics Section of the Iowa State Teachers' Association and the Mathematics Section of the Wisconsin State Teachers' Association.

The Executive Committee desires to thank all who have so heartily approved the Council and its plans. It also invites the support and cooperation of all other individuals and organizations who are yet on the outside. We desire to serve in the best possible way all teachers of mathematics who wish to cooperate with us.

C. M. Austin,
President National Council of Teachers of Mathematics.

Oak Park High School,
Oak Park, Ill.

THE WORK OF THE NATIONAL COMMITTEE ON MATHEMATICAL REQUIREMENTS.

The National Committee on Mathematical Requirements was organized in the fall of 1916 under the auspices of the Mathematical Association of America in response to an insistent demand that national expression be given to various movements looking towards reform in the teaching of mathematics which had gained more or less headway through the activities of various local organizations throughout the country. The Committee was instructed to investigate the whole field of mathematical education from the secondary school through the college and to make recommendations looking toward a desirable reorganization of courses and the improvement of teaching.

The original nucleus of the Committee consisted of six college professors who were appointed by Professor E. R. Hedrick, then President of the Mathematical Association. This group held its first meeting in September, 1916, at Cambridge, Mass., at which time steps were taken to secure adequate representation of secondary school interests and general plans for the work were formulated. The three large organizations of secondary school teachers of mathematics, viz., the Association of Teachers of Mathematics in New England, the Association of Teachers of Mathematics in the Middle States and Maryland and the Central Association of Science and Mathematics Teachers, were each requested to appoint an official representative on the Committee. Since then four additional representatives of secondary schools have been appointed.*

* The present membership of the Committee consists of the following: A. R. Crathorne, University of Illinois; C. N. Moore, University of Cincinnati; E. H. Moore, University of Chicago; D. E. Smith, Columbia University; H. W. Tyler, Massachusetts Institute of Technology; J. W. Young, Dartmouth College (chairman); W. F. Downey, English High School, Boston, representing the Association of Teachers of Mathematics in New England; Vevia Blair, The Horace Mann School, New York City, representing the Association of Teachers of Mathematics in the Middle States and Maryland; J. A.

It soon became apparent to the Committee that it could not hope to do its work with the necessary thoroughness or effectiveness without adequate financial support. Furthermore, the war naturally interfered with its work. Many of its members were engaged in war work of one kind and another and the remaining members were carrying a double burden in view of the fact that all educational institutions were greatly undermanned during this period. In the early spring of 1919, however, the Committee was fortunate in interesting the General Education Board of New York City in its work. As a result the Board appropriated the sum of $16,000 for the use of the Committee for one year ending July 1, 1920, and appropriated the further sum of $25,000 to complete the work.

This generous support made it possible to plan the work of the committee on a much larger scale. It made it possible to secure the full-time service of the chairman and vice-chairman; to rent and equip suitable offices; to employ the necessary stenographic and other clerical help; to pay the traveling expenses of members in attending meetings of the committee, etc. It also made it possible to finance numerous meetings of subcommittees, personal conferences and the like, and to pay the necessary expenses of representatives of the committee in attending and addressing meetings of various teachers' associations throughout the country. It is probably true that this is the first committee engaged upon a specific educational problem which has had adequate financial resources.

It was felt very strongly that the committee should not attempt to do the work assigned to it alone but that it should take steps to secure the active cooperation of teachers of math-

Foberg, Crane Technical High School, Chicago (vice-chairman), representing the Central Association of Science and Mathematics Teachers; A. C. Olney, Commissioner of Secondary Education, Sacramento, California; Raleigh Schorling, The Lincoln School, New York City; P. H. Underwood, Ball High School, Galveston, Texas; Eula A. Weeks, Cleveland High School, St. Louis, Mo. (Professor C. N. Moore was appointed to take the place of Professor Oswald Veblen, of Princeton University, who felt called upon to resign as a member of the Committee on account of his war duties Mr. G. W. Evans, of the Charlestown High School, Boston, Mass., was originally appointed as the representative of the New England Association. Owing to his absence from the country for over a year, his place was taken by Mr. Downey.)

ematics and school administrators throughout the country. This seemed necessary both for the purpose of securing their advice and constructive criticism and also in order to stimulate widespread interest and support so that the recommendations of the committee might be put into practice with the least possible delay. The attempt was therefore made to organize a truly nation-wide study and discussion of the problems of mathematical instruction. The response secured in this attempt was very gratifying. At the present time nearly one hundred organizations of teachers are cooperating with us.* Many of these have appointed special committees for this purpose. Our reports are submitted to these cooperating committees in preliminary form for their approval, comment, criticism and advice. The Committee is therefore acting in a very real sense as a national clearing-house for ideas relating to its problems. As a result it may confidently be expected that our final recommendations will have the support of the great majority of progressive teachers throughout the country; in fact, one of the most encouraging features of the Committee's work up to the present has been the very general agreement that has been found to exist among teachers. There is, of course, some difference of opinion as to details; but as to broad general lines of purpose and policy there is substantial unanimity.

It has been possible to secure wide publicity. Our reports have been presented and discussed at meetings of all the organizations previously referred to. "Notes and News" relating to our work have appeared at frequent intervals in some fifty educational periodicals. Above all, however, we are indebted to Commissioner P. P. Claxton and the U. S. Bureau of Education for the assistance given us by the publication and wide distribution of our reports. This service has been of inestimable value.

As to the present status of the work of the Committee, a list of reports, published or in preparation, with brief indications of their content, should be of interest.

The first five reports are intended to cover the field assigned

* The list of cooperating organizations includes 31 state teachers' associations, 25 associations and societies of teachers of mathematics (or of mathematics and science), local councils or clubs in 20 cities, and others.

to the Committee as to content and organization of courses. The first two have already been published.

1. "*The Reorganization of the First Courses in Secondary School Mathematics*," a preliminary report, published by the U. S. Bureau of Education as Secondary School Circular No. 5, February, 1920.

2. "*Junior High School Mathematics*," a preliminary report published by the U. S. Bureau of Education as Secondary School Circular No. 6, July, 1920.

The first of these considers the reorganization of courses in mathematics in the first two years of the standard four-year high school; that is, courses for the ninth and tenth grades. The second one considers the same problem for the junior high school; that is, for grades seven, eight and nine. The following principle is made fundamental in both reports:

> The primary purposes of the teaching of mathematics should be to develop those powers of understanding and analyzing relations of quantity and of space which are necessary to a better appreciation of the progress of civilization and a better understanding of life and of the universe about us, and to develop those habits of thinking which will make these powers effective in the life of the individual.

In the first report the following principle is also made basic:

> The courses in each year should be so planned as to give the pupil the most valuable mathematical information and training which he is capable of receiving in that year, with little reference to the courses which he may or may not take in succeeding years.

In the Junior High School Report, however, it is recommended that the work for the three years should be planned as a unit, in view of the fact that it may reasonably be expected that pupils will remain in the junior high school to the end of the ninth grade.

The application of these principles makes necessary a thoroughgoing reorganization of the content of courses. All topics, processes, and drill which do not directly contribute to the development of the powers mentioned in the first principle should, of course, be eliminated from the curriculum, and new material more appropriate to the purposes referred to should be introduced. In carrying out this application, the two reports contain detailed outlines as to topics which should be

included and as to the point of view which should govern the instruction. These reports have already been widely distributed and any further comment at this time would seem unnecessary.

3. "*Elective Courses in Mathematics for Secondary Schools.*"

This subject is in the hands of a subcommittee under the chairmanship of Professor C. N. Moore. The report, which is expected to be ready for publication in February, will consider the problem of the most desirable courses that should be offered by departments of mathematics in the later years of the secondary school, in particular in the last two years of the standard four-year high school and in the three years of the senior high school.

4. "*Mathematics in Junior Colleges.*"

This subject is in the hands of a subcommittee under the chairmanship of Mr. A. C. Olney, Commissioner of Secondary Education, California. The report is expected to be ready next spring.

5. "*The Function Concept in Secondary School Mathematics*," by E. R. Hedrick, University of Missouri.

In order to develop the "power of understanding and analyzing relations of quantity and of space" and the desired "habits of thinking," it is essential that the notion of the dependence of one quantity on another, *i.e.*, the notion of "functionality," be made fundamental in all mathematical instruction. Since this involves a radical departure from current practice, the Committee's recommendation that "the notion of relationship between variables be made the primary and underlying principle" of all courses in mathematics has called forth perhaps the greatest amount of comment. It seemed to the Committee desirable, therefore, to devote a special report to this topic, which would furnish teachers rather detailed information as to how this recommendation may be put into practice. The report will be ready for publication in January.

6. "*College Entrance Requirements in Mathematics.*"

The subject of this report, while obviously one with which the Committee would have to concern itself in any case, was deemed of such vital importance that the Councils both of the

Mathematical Association of America and of the American Mathematical Society specifically requested the Committee to take it up. The report gives the results of a rather extensive investigation into the needs of college departments, other than mathematics, regarding the mathematical preparation desired by them for students taking their elementary courses. On the basis of this investigation new definitions of entrance requirements in algebra have been formulated. It is gratifying to note that the problem of adequate preparation for college work as disclosed by the investigation mentioned does not conflict in any way with the recommendations made regarding the reorganization of courses in mathematics for secondary schools, which recommendations were formulated without any reference to college entrance requirements. In connection with the definition of the requirement in plane geometry a list of fundamental theorems and constructions has been prepared for the guidance of teachers and of examiners. This report has already been sent to the U. S. Bureau of Education for publication.

7. "*The Standardization of Terminology and Symbolism in Elementary Mathematics.*"

A report on this subject has been prepared by a subcommittee under the chairmanship of Professor David Eugene Smith, of Columbia University. It will be ready for publication in January.

8. "*The Present Status of Disciplinary Values in Education,*" by Vevia Blair, of the Horace Mann School, New York City.

This report will probably be ready for publication in February. It gives a critical review of all of the literature relating to the scientific investigation of the problem of disciplinary values and the transfer of training. On the basis of this review are formulated a number of propositions regarding transfer which seem to be justified by the experiments. These propositions have been submitted to leading psychologists in the country and have received the endorsement of the great majority of them. This report ought to be of great value in helping to bring order out of the chaos that has existed regard-

ing this subject for many years. It is obvious that the report has a large general educational significance and is not of interest merely to teachers of mathematics.

9. "*A Critical Study of the Correlation Method as Applied to Grades*," by A. R. Crathorne, University of Illinois.

Attempts to use the statistical method of correlation in the study of educational problems have been fairly numerous. The results in general have appeared, however, to be inconclusive. It appeared to the Committee distinctly worthwhile, therefore, to make a critical investigation of this method. To this end, Professor Crathorne undertook for the Committee the investigation of the correlations existing between a large number of pairs of subjects in the secondary school curriculum. In connection with this work nearly a thousand correlation coefficients have been computed. It is too early as yet to evaluate the results secured. They are being studied by a subcommittee consisting of Professor Crathorne, Professor C. N. Moore and Mr. Raleigh Schorling. Their report may be expected early in February.

10. "*Mathematics in Experimental Schools*," by Raleigh Schorling, The Lincoln School, New York City.

The recommendations of the Committee are based not on theoretical considerations alone but also on the actual experience of teachers throughout the country. It seems to the Committee that this experience should be made available to all teachers. Mr. Schorling was therefore requested by the Committee to prepare an extended report on the subject of the teaching of mathematics in experimental schools throughout the country. After a careful survey of the field, fourteen schools were selected as worthy of detailed consideration. Mr. Schorling's report will tell precisely what each of these fourteen schools is doing in the teaching of mathematics. It will give details as to the content of the curriculum, the equipment of the school, the training of the teachers employed, the cost of the instruction, the quality of students in the classes, etc. It is hoped that this report will be ready for publication in February or March.

11. "*World Experience Regarding Mathematical Curricula*," by J. C. Brown, State Normal School, St. Cloud, Minnesota.

This is an abstract of the report prepared on a similar subject by President Brown for the International Commission. It will be published as part of the Committee's final report.

12. "*The Training of Teachers*," by R. C. Archibald, Brown University.

Fundamentally the problem of better teaching must rest on the securing of better teachers. One of the very real obstacles in the way of reform in the teaching of mathematics lies in the fact that comparatively few teachers of this subject at the present time are adequately trained. Professor Archibald, who is already well-known for his report on the same subject which he prepared for the International Commission on Mathematics, is now making a very careful investigation of this subject for the Committee. His report will discuss actual conditions in all the states of the union and in a considerable number of the larger cities; will give information regarding the courses primarily intended for teachers given in colleges, universities, and normal schools and will make definite recommendations regarding such courses. It is hoped that this report will be ready for publication in February.

13. "*The Interests of High School Students*," by W. F. Downey, English High School, Boston, Mass.

The object of this investigation was to obtain from high-school pupils a statement regarding their interests and preferences in the study of mathematics. The schools selected for this investigation were of different types and sizes and geographically widely separated. In order to obtain an honest expression of opinion from the pupils, the replies were in writing without signature and those who conducted the inquiry in each school were asked to impress upon the pupils the purpose of the investigation. Replies were received from about 7000 pupils of fifteen high schools. In reply to the question "Do you like mathematics?" 84 per cent. answered in the affirmative. The investigation concerned itself further with the question as to what topics were liked most and the reasons therefore. Also

as to which subjects in the high-school curriculum were liked best. This investigation has been completed for some time but has not as yet been published.

14. "*Change of Mind between High School and College as to Life Work*," by A. R. Crathorne, University of Illinois.

Much of the agitation for the early introduction of vocational mathematics rests on the assumption that a considerable proportion of high-school pupils reach a decision early in their course as to what their life work is to be. It seemed desirable, therefore, to make an effort to find out to what extent this assumption is justified. Replies to a questionnaire were secured from over 2,000 college freshmen from eleven widely scattered states. Of these students, 57 per cent. had entered high school with some definite occupation in view. On entrance to college about one half of these had changed their minds. A summary of this investigation was published in *School and Society*, vol. XI, No. 262 (Jan. 3 ,1920), pp. 28–30, and the report in full was published in *Educational Administration and Supervision*, vol. VI, No. 5 (May–June, 1920), pp. 274–284.

15. "*The Importance of High School Mathematics as Indicated by Certain Questionnaires*," by Alfred Davis, Soldan High School, St. Louis, Mo.

A number of questionnaire investigations of interest to teachers of mathematics have recently been made and published. The Committee requested Mr. Davis to gather together in one report the results secured. His report includes, furthermore, an investigation made by himself but not hitherto published.

16. "*Bibliography*."

A bibliography on the teaching of mathematics covering all articles in a selected list of leading mathematical and educational journals since 1910 has been carefully prepared with the assistance of the graduate students in Professor Smith's classes at Teachers College, Columbia University. This bibliography will give not only the authors, titles and place of publication of the various articles but will also give brief synopses of them. It is expected that this report will be ready for publication in January.

A number of other projects are in various stages of preparation. Provost C. B. Upton of Teachers College, Columbia University, is engaged upon a report for the Committee which is intended to make available to teachers of mathematics information regarding various standardized tests which should be of value in connection with the teaching of mathematics. This informationn is at present widely scattered and it is believed that a real service will be done by bringing it together in convenient form. The Committee has further suggested the initiation of various investigations into the mathematical elements entering into the various professions, industries, occupations, etc. A memorandum on this subject has been sent to the various cooperating organizations, graduate schools of education, and the like, in the hope that some individuals or groups of individuals will see their way clear to undertaking such investigations which, if carried through in a thoroughly scientific spirit, would obviously be of great value.

This brief survey of the work of the Committee indicates that its various reports will all be completed by next spring. The remaining few months of its existence (until July 1, 1921) will be devoted to the revision of those reports which were of a preliminary character and the organization of the whole series into a single final report, if means can be found for its publication in a single volume.

The thought at once suggests itself, however, as to what agency will carry on the work initiated by the Committee. After all, the publication of our final report marks the beginning, not the end, of the real work. Who will see to it that our recommendations are put into practice? Who will observe the results and make the cumulative experience available for the guidance of all in further progressive improvement? How are adequately trained teachers to be secured in sufficient numbers? These are only a few of the many questions that thrust themselves on the consciousness of any one interested in the problem of placing the teaching of mathematics in a position of maximum service in the education of our citizenship.

The hundreds of teachers who have taken an active part in the work of the Committee during the last year and a half—and the thousands of others who have followed this work with

interest and hope—may surely be counted on to do all in their power to see to it that this work, begun under unusually favorable circumstances, is carried on with increased effectiveness in the years to come. It seems clear in the light of the experience of the last eighteen months that one of the things needed is nation-wide organization and another is energetic leadership. It is earnestly to be hoped that such organization and leadership can be found. Perhaps the National Council of Teachers of Mathematics has come into being at just the right time to help to carry this movement forward. It can almost certainly do so, if it receives the necessary support on the part of the teachers of mathematics throughout the country. There are probably some thirty thousand such teachers. If all of them, or one half of them, or even one fourth of them, were effectively organized and were ready to give a little of their money as well as their time to the cause, their influence for good, both to themselves individually, to their profession, and to their students, would be limited only by their wisdom and their enthusiasm.

J. W. Young,
Chairman, National Committee on Mathematical Requirements.

Dartmouth College.

A SYMPOSIUM OF DISCUSSION ON THE NATIONAL COMMITTEE REPORT ON JUNIOR HIGH SCHOOL MATHEMATICS.

For the convenience of the reader, parts IV, V, and VI of the National Committee Report on Junior High School Mathematics are reprinted here. The complete report, *Secondary School Circular No. 6*, July, 1920, may be secured from the U. S. Bureau of Education. Following the reprint are five discussions of the report.

IV. General Considerations Concerning the Mathematics of the Junior High School.

1. The primary purpose of the teaching of mathematics should be to develop those powers of understanding and analyzing the interdependence of quantities and spatial magnitudes which are necessary to a better appreciation of the progress of civilization and a better understanding of life and of the universe about us, and to develop those habits of thinking which will make these powers effective in the life of the individual.

2. All topics, processes, and drill entering into the course must be justified with reference to the development of the powers of understanding and analyzing relations of quantity and space, and to the development of those habits of thinking, that have just been referred to.

3. At the end of the sixth school year the pupil should be able to perform the fundamental operations with integers and with common and decimal fractions with accuracy and a fair degree of speed.

4. In view of the fact that pupils may be expected to remain in school until the end of the junior high school period, instead of leaving, in large numbers at the end of the eighth school year, as at present, the mathematics if the three years of the junior high school should be regarded as a unit.

5. The course should include arithmetic, intuitive geometry,

algebra, numerical trigonometry, and an introduction to demonstrative geometry.

6. The dependence of one quantity upon another, which involves the idea of function, should be emphasized throughout.*

7. Advantage should be taken of the opportunities afforded in the solution of problems for the exercise of judgment and self-reliance, and for the development of accuracy and a fair degree of speed in computation.

8. Emphasis should be placed on the use of common sense in computing from approximate data; as, for example, to fix the idea that the area of a circle can not be found accurately to the thousandths of a square inch, if the measurement supplies only the number of whole inches in the diameter. Pupils should be encouraged to use interest tables, tables of squares, square roots, etc., in computation.

9. The systematic development by observation, measurement, and construction of the elementary properties and relations of geometric figures should lead gradually to methods of abstract and formal proof.

10. The applications of arithmetic to business should be continued late enough in the course to bring to their study the pupil's greatest maturity, experience and mathematical knowledge, and to insure real significance of this study in the business and industrial life that many of the pupils will enter upon at the close of the junior high school period.

11. The teachers needed for the mathematics classes of the junior high school can probably be secured most advantageously by selection from the ranks of successful teachers of the upper grades of the elementary school. These selected teachers should undergo a course of training in the subject matter and methods of high-school mathematics before entering upon junior high-school teaching. In the future it will be highly desirable to require special training for teachers preparing to teach in the junior high school. It should be clear that the work will not attain the standard desired unless the teacher knows the subject of high-school mathematics thoroughly and sees the need for vitalizing the work and relating it to the interests and experiences of the pupils.

* A special report on the rôle of the function concept in secondary school mathematics will be published in the near future.

12. Arithmetic should not be completed before the pupil has acquired the power of using algebra as an aid.

13. Intuitive geometry should be introduced before algebra is taken up as a general topic, although algebraic processes should be introduced as needed in the work in mensuration.

14. Numeral trigonometry should be based upon algebra and intuitive geometry, rather than upon demonstrative geometry.

15. It is desirable that each pupil should, upon leaving school, know the significance of a demonstration in geometry. Such demonstrative geometry as is offered may properly come rather late in the course, since it requires more intellectual maturity than is needed in the other work.

V. Material of the Course in Mathematics for the Junior High School.

In the course in mathematics for the junior high school outlined in the following pages no attempt is made to prescribe the order of presentation of the topics included. Much experimentation must precede any formulation of content and order that can make any claims to validity and authority.

The following recommendations as to content and arrangement of the course of study are intended to form the basis of study, discussion, and classroom experiment. With the cooperation of the supervisors of junior high schools, and their mathematics teachers, serious and constructive criticism, based on actual classroom experience with junior high-school pupils, may be gathered by the National Committee on Mathematical Requirements, and made available to all interested in the development of the junior high school. The outcome of this cooperation, with the national committee serving as a clearing house of ideas and material, should be highly significant for the teaching of mathematics in this type of school.

It is felt that this experimentation will not produce the best results if this committee should attempt, at the present time, to specify, except in a very general way, the topics to be considered in the several school years (see Sec. VI of this report).

Stated by topics rather than years, the mathematics of the junior high school may properly be expected to include the following:

A. Arithmetic:

(*a*) The fundamental operations of arithmetic.

(*b*) Tables of weights and measures in general practical use.

(*c*) Emphasis on simple fractions: $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{5}$, $\frac{1}{8}$. Fractions other than these to have less attention.

(*d*) Facility and accuracy in the four fundamental operations; time tests, taking care to avoid subordinating the teaching to the tests, or to use the tests as measures of the teacher's efficiency.

(*e*) Very simple short cuts in multiplication and division (such as replacing multiplication by 25 by multiplying by 100 and dividing by 4).

(*f*) Percentage. Interchanging common fractions and per cents., finding any per cent. of a number, finding what per cent. one number is of another, finding a number when a certain per cent. of it is known; such applications of percentage as come within the student's experience.

(*g*) Line, bar, and circle graphs to be used wherever they can be used to advantage; these not to be taught as a separate topic.

(*h*) Arithmetic of the home: Household accounts, thrift, simple bookkeeping, methods of sending money, parcel post.

Arithmetic of the community: Property and personal insurance.

Arithmetic of civil life: Taxes.

Arithmetic of banking: Savings accounts, checking accounts, foreign money.

Arithmetic of investment: Real estate, elementary notions of stocks and bonds, postal savings.

(*i*) Statistics. Fundamental concepts; statistical tables and graphs; pictograms; simple frequency distributions; measures of central tendency.

B. Intuitive Geometry:

(*a*) The direct measurement of distances and angles by means of a linear scale and protractor. The approximate character of measurement. The degree of precision as expressed by the number of significant figures.

(*b*) Indirect measurement by means of drawings to scale. Uses of square ruled paper.

(*c*) Areas of the square, rectangle, parallelogram, triangle, and trapezoid; circumference and area of a circle; surfaces and volumes of solids of corresponding importance.

(*d*) Practice in numerical computation with due regard to the relation between the precision of the data and the significant figures in the result.

(*e*) Simple geometric constructions with ruler and compasses, T-square and triangle, such as perpendicular bisector, angle bisector, parallel lines, etc.

(*f*) Familiarity with such forms as the equilateral triangle, the 30°–60° right triangle, and the isosceles right triangle; symmetry, axial

and central; a knowledge of such facts as those concerning the angle sum for the triangle and the Pythagorean relation; simple cases of geometric loci in the plane and in space.

(*g*) Geometry of appreciation: Geometrical forms in nature, architecture, manufacture, and industry.

The work in intuitive geometry should make the pupil familiar with the elementary ideas concerning geometric forms in the plane and in space with respect to shape, size, and position. It should, moreover, be carefully planned so as to bring out geometric relations and logical connections. Before the end of this intuitive work the pupil should have very definitely begun to make inferences and draw valid conclusions from the relations discovered. In other words, this informal work in geometry should be so organized as to make it a gradual approach to, and provide a foundation for, the subsequent work in formal demonstrative geometry.

C. Algebra:

1. The formula—its construction, meaning, and use—
   (*a*) As a concise language.
   (*b*) As a shorthand rule for computation.
   (*c*) As a general solution.
   (*d*) As an expression of the dependence of one variable on other variables.

The work done with the formula will include translation from English into algebraic language, and vice versa.

2. Graphs and graphic representations in general—their construction and interpretation in—
   (*a*) Representing facts (statistical, etc.)
   (*b*) Representing dependence.
   (*c*) Solving problems.
3. Positive and negative numbers—their meaning and use—
   (*a*) As expressing both magnitude and one of two opposite directions or senses.
   (*b*) Their graphic representation.
   (*c*) The fundamental operations applied to them.
4. The equation—its use in solving problems—
   (*a*) The linear and the "pure" quadratic equation in one unknown; their solutions and applications.
   (*b*) Equations in two variables, with numerous concrete illustrations.
   (*c*) A simple treatment of proportion. To include various simple applications of ratio and proportion in cases in which they are generally used in problems of ordinary life. In view of the usefulness of the ideas and training involved, this subject may also properly include simple cases of variation.
5. Algebraic technique:
   (*a*) The fundamental operations.

Their connection with the rules of arithmetic should be clearly brought out and made to illuminate arithmetical processes. Drill in these operations should be limited strictly in accordance with Principles 1 and 2, mentioned in

Section IV above. In particular, nests of parentheses should be avoided, and multiplication and division should not involve much beyond monomial and binomial multipliers, divisors, and quotients.

(*b*) Factoring. The only cases that need be considered are—

(i) Monomial factors.

(ii) The difference of two squares.

(iii) The square of a binomial.

(*c*) Fractions.

Here again the intimate connection with the corresponding processes of arithmetic should be made clear and should serve to illuminate such processes. The four fundamental operations with fractions should be considered only in connection with simple cases, and should be applied constantly throughout the course to gain the necessary accuracy and facility. The most difficult complex fractions taken up should contain only numerical fractions in numerator and denominator.

5. Algebraic technique—Continued.

(*d*) Exponents and radicals. The work done on exponents and radicals should be confined to the simplest material required for the treatment of formulas.

(*e*) Stress should be laid upon the need for checking solutions.

(*f*) Optional topics. Logarithms and the slide rule may be taken up with some classes. When either of these topics is included, it is recommended that discussion of the underlying theory be omitted. The subject may be taken up in connection with arithmetic or trigonometry.

D. Numerical Trigonometry:

(*a*) Definition of sine, cosine, and tangent.

(*b*) Their elementary properties as functions.

(*c*) Their use in solving problems involving right triangles.

(*d*) The use of tables of these functions (three or four places).

The introduction of the elementary notions of trigonometry into the earlier courses in mathematics has not been as general in the United States as in foreign countries. Among the reasons for the early introduction of this topic are its great practical usefulness for many citizens; the insight it gives into the nature of mathematical methods, particularly those concerned with indirect measurement; the rôle that mathematics plays in the life of the world; the fact that it is not difficult and that it offers wide opportunity for concrete and significant application; and the interest it arouses in the pupils.

E. Demonstrative Geometry.

The demonstration of a limited number of propositions, with no attempt to limit the number of fundamental assumptions, the sole purpose being to show to the pupil what "demonstrations" means.

Many of the geometric facts previously inferred intuitively may be used as the basis on which the demonstrative work is built. This is not intended to preclude the possibility of giving at a later time rigorous proofs of some of the facts inferred intuitionally. It should be noted that from the strictly logical point of view the attempt to reduce to a minimum the list of axioms, postulates, or assumptions is not at all necessary, and from a pedagogical point of view such an attempt is very undesirable. It is necessary, however, that those propositions which are to be used as the basis of subsequent formal proofs be explicitly listed and their logical significance recognized.

VI. Suggested Arrangements of Material.

It is not the intention of the committee to suggest a single course of study either by years or by half years, or to recommend any artificial separation of topics; nevertheless, it is felt that a few possibilities in the arrangement of material may be helpful to teachers in deciding upon a course that is suited to their needs. The following plans for the distribution of time are therefore suggested, but no one of them is recommended as superior to the others, and only the large divisions of material are mentioned.

Plan A.

First year: Applications of arithmetic, particularly in such lines as relate to the home, to industry, to thrift, and to the various school subjects; intuitive geometry.

Second year: Algebra; applied arithmetic, particularly in such lines as relate to the commercial, industrial, and social needs of our country.

Third year: Algebra, trigonometry, demonstrative geometry. (By this plan the demonstrative geometry is introduced in the third year and arithmetic is practically completed in the second year.)

Plan B.

First year: Applied arithmetic (as in Plan A); intuitive geometry.

Second year: Algebra, intuitive geometry, trigonometry.

Third year: Applied arithmetic, algebra, trigonometry, demonstrative geometry.

By this plan trigonometry is taken up in two years, and the arithmetic is transferred from the second year to the third year.

Plan C.

First year: Applied arithmetic (as in Plan A); intuitive geometry; algebra.

Second year: Algebra; intuitive geometry.
Third year: Trigonometry; demonstrative geometry; applied arithmetic.
By this plan algebra is confined chiefly to the first two years.

Plan D.

First year: Applied arithmetic (as in Plan A); intuitive geomtery.
Second year: Intuitive geometry; algebra.
Third year: Algebra; trigonometry; applied arithmetic.
By this plan demonstrative geometry is omitted entirely.

Plan E.

First year: Intuitive geometry; simple formulas; elementary principles of statistics; arithmetic (as in Plan A).
Second year: Intuitive geometry; algebra.
Third year: Intuitive geometry; numerical trigonometry; arithmetic.
By this plan the work of the first two and one-half years may be described as general mathematics, while the last half-year would be devoted to the special civic and economic features of business practice.

Discussion of Part IV.

There was never an opportunity to work out a course in secondary mathematics as is now offered in the junior high-school movement. Assuming only the fundamental operations of arithmetic as prerequisites, with no traditions to interfere with the formation of the course, we are entirely free to select what material seems best fitted for our purposes,

As the primary purpose of the teaching of secondary mathematics the committee states "the development of the powers of understanding and analyzing the interdependence of quantities and spatial relations which are necessary to a better appreciation of the progress of civilization and a better understanding of life and the universe about us and to develop those habits of thinking which will make these powers effective in the life of the individual." Furthermore, the committee urges teachers "to take advantage of the opportunities for training judgment and self-reliance and for developing accuracy and a fair degree of speech in computation." Nothing is to be included in the course which cannot be justified on the basis of these aims.

It must be admitted that the success of a course in mathematics is directly proportional to the extent to which it succeeds in accomplishing these aims. They are the chief reasons why mathematics has held successfully a place on the curricu-

lum. However, these porposes cannot be accomplished to the fullest extent unless the material included in the course is also justified on the basis of real needs in the life and studies of the adolescent boy or girl, and fits them to perform in later life the activities in the affairs of an active world. For, these performances not only call for the use of the indirect outcome of the study of mathematics, such as habits of thinking, and powers of analysis, generalization and concentration, but for direct use of mathematics in present-day social life and business.

Moreover, all material which can not be adapted to the mental development of the pupils, or lies outside of their experiences is of little or no value and should be rejected.

The committee suggests that the new course include further arithmetic, intuitive geometry, algebra, numerical trigonometry, and an introduction to demonstrative geometry. The following supplementary suggestions are offered.

1. The applications of arithmetic should be continued to be taught late in the three-year course.

2. Intuitive geometry is to be taken up early.

3. Algebraic processes and arithmetic should be studied in connection with mensuration.

4. There should be some numeral trigonometry.

5. Some demonstrative geometry might be introduced late in the course.

This material should prove to be far better than the traditional courses in arithmetic usually offered in the seventh and eighth grades. Recent experiments tend to show that pupils of that age can do sucessfully much of the work now taught in the senior high school, while they profit little by studying many of the applications of arithmetic because they are beyond their experience.

Although correlation is not advocated by the committee, the second, third and fourth points indicate the possibility, or even the desirability, of correlating algebra, geometry, arithmetic, and trigonometry. The plans of alternating these subjects after certain time intervals, or of carrying them parallel to each other, are usually not successful. The work ought to be so arranged that there is a motive, which the pupil can recog-

nize for changing from one topic to another. Since the teacher, busy with classes and other school duties, is not in a position to put the material in the proper order, there is need for a course in which the topics are so arranged that they may be studied by a class without interruptions. The report does not show that the committee have considered the plan, at least not sufficiently to include it as one of the possible arrangements worked out in part VI of the report.

Such a course should accomplish the aims and purposes set up by the committee more easily than is possible with separate courses in arithmetic, algebra, and geometry. Topics could then be put in psychological order with the result that pupils would gain from the study of mathematics the broadest possibilities.

Correlation is rapidly growing not only in secondary mathematics but also in the junior colleges, and the committee should take a definite stand in this movement.

Besides the general aims set up by the committee the report mentions several specific aims. One of these is the emphasizing of the idea of function throughout the course. Functional relations occur even in some of the simplest applications of arithmetic. We find them in problems of cost, interest, percentage. Many of the principles of geometry express functional relations. These relations are expressed in tables, in symbols as formulas, or by graphs. They should be felt and understood by the pupils long before the term function is mentioned to them. It is a concept which takes on more and more meaning as the pupil's knowledge is enriched, and as he passes through the study of secondary mathematics on to courses in college algebra and analytical geometry. If the idea of function is emphasized thorughout the courses in the junior high school, this should be extremely beneficial to the more advanced courses, indeed it should form a powerful motive to induce the pupil to study advanced work.

Another specific aim mentioned in the report is "emphasis on the use of common sense in computing from approximate data." Even a careless examination of textbooks in arithmetic and trigonometry reveals how little attention is paid to this important subject. Answers given in problems in which

the original data have scarcely two-figure accuracy, are computed to five and six figures by means of trigonometric tables. In computations of areas and circumferences of circles in which the radius is expressed to one or two figures we find frequently that a value of π is used with three or four decimal places. In all problems of mensuration the pupil should be led to consider whether or not the result is reasonable. Abbreviated multiplication and division should therefore be taught in the seventh grade until every pupil completely masters these processes.

The question of finding the right teachers for the junior high school is an important one. High school teachers are usually prepared to do this work as far as knowledge of subject matter is concerned. They have the broad outlook to teach the material contained in the course, but few have had sufficient experience with pupils of this age. A thorough understanding of this critical period of child life is absolutely necessary. On the other hand, most elementary school teachers lack the necessary academic training, and do not have the advanced viewpoint. The committee is right in advocating that a new teacher be trained, most advantageously selected from the upper grades of the elementary school, to teach the mathematics classes of the junior high school.

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Discussion of Part V.

The Committee on Mathematical Requirements is to be congratulated on its recent Preliminary Report on Junior High School Mathematics.

In Section V, suggested topics are listed under the headings of Arithmetic, Intuitive Geometry, Algebra, Numerical Trigonometry, and Demonstrative Geometry. These headings are used evidently for convenience, not with the idea that the material should be presented as such separate branches. Although it is expected that this work will be fused into a single course in Junior High School Mathematics, the general trend will be in the sequence given. That is, in the first part of the course Arithmetic will be stressed. Intuitive Geometry should come

before Algebra, and Numerical Trigonometry before Demonstrative Geometry. But, certain topics in Arithmetic are best taught through Intuitive Geometry, as the tables of weights and measures [A. (b)–]. The equation in Algebra is best introduced through the formulas of mensuration. The indirect cases of percentage [A. (f)–] must be taught algebraically by the equation either in its regular or camouflaged form.

Among the suggestions under Arithmetic, a few may well be stressed, as

1. Limitation of fractions to simple ones.
2. Time tests.
3. Teaching of simple bookkeeping.
4. Stressing civic and social aspects of mathematics.
5. Use of graphs only to illuminate other mathematics.

It may be opportune to give a word of warning not to overstress such topics as the indirect cases of percentage, foreign money, stocks and bonds, and statistics.

The outline of Intuitive Geometry is well thought out. The success of this work is phenomenal if it is presented so that it is really intuitive, nor formal. There is no other phase of elementary mathematics that makes such an appeal to all types of pupils.

With the Intuitive Geometry as a background, the phases of elementary algebra, as outlined, can be easily presented, for algebraic expressions come to have a real meaning, impossible otherwise. If teachers of Algebra will make it their first concern to see that algebraic symbols and expressions mean something to the pupils, they will have need of less concern about their manipulation. When pupils recognize quadratic trinomials as areas of squares or rectangles, and their factoring as finding the dimensions, a real achievement has been made.

The Committee wisely recommended the limiting of factoring to three cases, but I hardly agree on the three. Personally, I prefer limiting it to two cases,

(1) The monomial factors,
(2) The quadratic trinomial.

By teaching the general cross product method for all such expressions, no separate process need be taught for the square of a binomial or for the difference of two squares. The one method may be applied to all three types.

The introduction of numerical trigonometry into the mathematic course of the junior cycle is highly desirable. It is necessary for a complete survey of the elementary field. Its practical usefulness appeals to pupils immensely. It offers an opportunity for immediate, interesting, and concrete application. At the same time it is decidedly more within the scope of the pupils' ability than the abstruse topics of Algebra formerly included in a ninth grade course. As one boy, brought up under the old regime of formal arithmetic, algebra, geometry and trigonometry, said "I never saw any sense to mathematics until I got to trig. If they had given me that earlier, I might have liked mathematics and learned it."

To be complete the junior cycle should give at least a glimpse into demonstrative geometry, only to let the pupil know what it is. Intuitive geometry fuses perfectly with other mathematics. *Demonstrative geometry cannot, for it is really logis, not mathematics.* But enough of it should be given at the close of the ninth grade so that the pupil may know what it is and whether or not he wishes to elect it for further study in the senior high school.

The report suggests that "Many of the geometric facts previously inferred intuitively may be used as the basis on which the demonstrative work is built." This is true, but care should be taken to choose theorems in which the pupil sees something to prove, not self-evident facts or assumptions.

When all seventh, eighth and ninth grades, whether they are organized in junior high schools or not, follow the outline recommended by the Committee, a great forward step will have been taken in the teaching of mathematics.

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DISCUSSION OF PART V.

An objective discussion of the tentative syllabus embodied in Section V of the report may properly begin with an expression of appreciation and general approval. Its brevity and directness will undoubtedly be welcome to the majority of teachers. There is, throughout, a gratifying effort to secure perspective by putting the essentials into bold relief, and

to avoid the usual multiplicity of details found in other official syllabi.

Moreover, a commendable flexibility characterizes the recommendations of the National Committee. Thus, graphs are "to be used wherever they can be used to advantage." Applications of percentage are to be such "as come within the student's experience," and the customary minute specifications are omitted. In view of the prevailing educational uncertainty this latitude in the phraseology of the report is undoubtedly its great outstanding virtue, though only a temporary one. The National Committee will probably find it desirable to issue a series of supplementary reports dealing in detail with certain features of secondary mathematics which are under debate at present. There is as yet no definite national policy concerning intuitive geometry. Very few teachers understand why intuitive geometry should come early in the course, why it should precede algebra, and what methods are to be employed in teaching it. To the majority it is an uncharted sea. In arithmetic, such topics as "household accounts, thrift, simple bookkeeping, statistics," are receiving very different degrees of emphasis in the newer texts. The whole arithmetic situation is in a state of flux. Old landmarks are disappearing and the new horizon is not too distinct. Some of the recent demands in algebra, such as the insistence on the formula, on the function concept, and on more significant problem material, are still ignored in the average classroom, and even experienced teachers find it difficult to effect a satisfactory compromise between tradition and reform. Trigonometric material, even of the simplest type, is also *terra incognita* in most of our junior high schools.

The textbooks are approaching all these things with great moderation and caution. An impartial reader usually gets the impression that the newer ideas are introduced, if at all, as a concession, and that almost invariably they have no organic connection with the remainder of the text. In fact, they may even be omitted "at the discretion of the teacher," without in the least disturbing the sequence. Such slow and artificial revisions of the curriculum can hardly be our ultimate goal. Perhaps an individual author or publisher cannot be

expected to submit a radical teaching program. It would be too costly a venture. And since textbooks so largely determine the work of the teachers, a modification of the usual routine can come only through a national program of readjustment, combined with much experimentation. Once this program has found general recognition, authors may safely begin to evolve really new texts.

It is clear, therefore, that a mere enumeration of topics will hardly suffice for classroom purposes. Specific illustrations and suitable problems must be included eventually, at least in certain critical phases of the work. The National Committee undoubtedly shares this feeling, for it has announced the impending publication of a separate report on "The Function Concept in Secondary School Mathematics." There should also be reports on arithmetic, on intuitive geometry, and on problem material in algebra.

Another matter of importance will require attention at an early date. I refer to the approximate time allotment necessary to secure a fair degree of proficiency in the average classroom. Investigations show that a great discrepancy exists in the time schedules of the various junior high schools. Some have supervised study, without homework, while some permit home assignments of a very flexible character. The number of recitations varies from three to five a week, and the length of recitations is subject to extreme fluctuations. Such divergent conditions of work must produce a corresponding dissimilarity of content and methods. Hence there must be an authoritative evaluation of topics in terms of lesson units. Thus, a minimum of ten lessons of 40 minutes each should be reserved for positive and negative numbers. At this point the interpretation of the syllabus is dependent on the psychology of learning and on scientific experimentation. The inexperienced teacher will be helped immensely by authentic time schedules. Here, again, the National Committee will make a valuable contribution by its forthcoming report on "Mathematics in Experimental Schools." In general, the curriculum outlined by the National Committee, with the exception of demonstrative geometry, may be completed satisfactorily within the three years of the junior high-school course.

Finally, it cannot be denied that the fluctuations mentioned above are often due to mutually exclusive and conflicting theories concerning the aims of secondary mathematics. There are those, a minority perhaps, who see in mathematics *merely* a unique tool for mental discipline. Accordingly, "nests of parentheses," complex fractions, and other non-essentials, are treated with relentless and time-consuming thoroughness. On the other hand, now and then we are told with equal conviction that in the junior high school the pupil is to be "tried out," to be "exposed" to mathematics, so that he may get a general idea "what it is all about." Radically different results must accompany these antagonistic views. Any experienced teacher knows that a *cumulative* subject like mathematics becomes a veritable nightmare of inefficiency, and a torture to pupils and teachers, unless the fundamental technique is made practically automatic. A lecture or two on graphs may be instructive and interesting, but that is a very different thing from securing a functioning familiarity with the subject. In the subsequent remarks it is assumed that a reasonable mastery of the essentials is one of the aims endorsed by the National Committee. (See Section [illegible] of the report, especially statements 1, 2, 7.)

Special Considerations.

Arithmetic. In Section IV, 3, it is stated that "at the end of the sixth school year the pupil should be able to perform the fundamental operations with integers and with common and decimal fractions with accuracy and a fair degree of speed." If one may believe the current criticisms of our elementary schools, this desirable goal is rarely or never reached under existing conditions. The syllabus obviously sustains that opinion, for in Section V, *A*, topics (*a*), (*b*), (*c*), (*d*), (*e*), are devoted essentially to a review of the four fundamental operations. Concerning topics (*f*) and (*h*) one may quote almost verbatim Chapter III from the "Teaching of Arithmetic," by Professor D. E. Smith, especially the following passage:

"When we consider this list of topics (*i.e.*, the arithmetic topics previously enumerated by the author), we are struck

by its simplicity and brevity. There is little here to try unduly the understanding of a learner. When we think that this is all that the world usually demands of the school, and that we are allowed eight years to impart this knowledge, we are led to ask ourselves why the world is not satisfied with the results. Is the difficulty with ourselves in that we include a lot of matter of relatively little value, but which consumes the time without any just return? Or do we fail to insist on the fundamentals while we are teaching the more advanced topics that find place in our schools?"

Topics (*g*) and (*i*) are very valuable additions to the customary routine. They furnish a natural bond between arithmetic and geometry. One question is pertinent. "Business applications" dominate the arithmetic of the junior high school, perhaps justly so. But should we not begin to insist that the science of number is not inherently confined to dollars and cents? It can talk many languages. In fact, it is "encyclopedic." As the curriculum of the junior high school expands, why should not arithmetic assume a wider range by giving due attention to science and to household and industrial arts?

Intuitive Geometry. Topics (*a*), (*b*), (*c*), (*d*) are concerned with measurement. Experience shows that pupils in the seventh grade can readily measure segments "to the nearest millimeter." After some practice they can be made to read results within half a millimeter. That seems to be the limit of accuracy possible with the instruments available. Hence the remarks of the syllabus concerning the "approximate character of measurment" and concerning "significant figures" must be interpreted accordingly. As to topic (*e*), one may question the obligatory use of the "T-square and triangle." Many schools will find it difficult to furnish such an equipment at this time. There is ample opportunity for efficient construction work without the usual accessories of mechanical drawing. Symmetry, mentioned in (*f*), should be a tool and not merely a relational fact. Much stress should be laid on topic (*g*), so frequently ignored altogether.

The syllabus emphasizes the study and appreciation of geometric forms, measurement and computation, constructions,

and geometric relations. It does not offer suggestions as to arrangement or sequence. It might be well to inform inexperienced teachers that intuitive geometry is not primarily a textbook subject, that it requires constant contact with concrete material, and that the method of procedure is all-important.

Algebra. Perhaps the material outlined in the tentative syllabus may be summarized as follows: the equation, the graph, the formula, and the technique which is necessary to make algebra effective for purposes of computation and problem solving. Positive and negative numbers may be included under the heading of technique. This moderate outline is in harmony with modern demands. A few remarks concerning each of these main divisions may be appropriate.

1. *The equation.* Apparently the "complete" quadratic equation is to be omitted. That means the elimination of the "quadratic formula" from the elementary course. The wisdom of this suggestion depends entirely on the question whether one can find at least a few good, non-technical, applied problems that can be solved only by means of "quadratics." Equations in two variables are to be included, "with numerous concrete problems." It must be confessed that the problem material usually following these equations is not of very convincing importance. Proportion should be treated much more liberally, in view of its constant use in science and in the problems of ordinary life. This seems to be the principal opportunity to make "fractional" equations worth while. Moreover, the function concept in its simplest form finds a ready symbolic expression in the study of proportionality.

2. *The graph.* The representation of statistical data is ordinarily of a merely informational or preparatory character. The truly mathematical use of graphs is concerned with the picturing of functional relations and with the solution of problems. Statistical (non-mathematical) graphs have finally invaded the elementary books, but functional graphs thus far seem to exist chiefly for ornamental purposes.

3. *The formula.* There are those who look upon this topic as the one great justification of algebra in an elementary

course. In the shop, and in the literature intended for the general reader, the formula is said to be the only form of algebraic training found universally necessary. However that may be, for school room purposes this contention is subject to two very serious difficulties. The transformation of even ordinary formulæ often requires a respectable amount of technique not to be acquired in a moment. And the manipulation of formulæ which are meaningless to the pupil is as objectionable a bit of formalism as was the juggling of the one-sided technique of former days. The remedy must eventually be found by furnishing data from which the pupil can *derive* the underlying relations. These relations, expressed in graphic or symbolic form, can then furnish a suitable point of departure for a subsequent extension to discoveries made by others. This is training in functional thinking, so persistently demanded by the world's leading mathematicians. A good beginning in this direction has been made. (See Nunn, The Teaching, of Algebra, Chapter III. Also, Nunn, Exercises in Algebra, Part I.)

4. *Algebraic technique.* The reduction of factoring to three "cases" will be welcome to those who have long regarded with distress the really alarming amount of time that was generally found necessary for this barren field. The omission of the general trinomial is a logical sequel to the disappearance of complete quadratics. Naturally, the usual work in fractions will then be shortened very considerably. In fact, the chief use of algebraic fractions will probably center around the transformation of formulæ. The (optional) introduction of logarithms is perhaps the only means of ever making exponents seem significant to a pupil.

As was stated previously, unless the formula is to become the backbone of elementary algebra, a more promising collection of problems must be forthcoming. The National Committee may find the creation of a vitalized algebra course suitable for the average pupil in the junior high school, a domain of investigation of which only the general outlines have been discerned.

Numerical Trigonometry. This enrichment of the curriculum will occasion very little difficulty. Elementary trigo-

nometric notions may be introduced at any time, even as early as the seventh grade. The chief precautions should be to "make haste slowly." One of the principal contributions of even a rudimentary treatment of trigonometry will be the immense increase in the range of significant problems.

Demonstrative Geometry. There are a few good American schools, usually private or endowed, that succeed in doing some excellent work in demonstrative geometry in the ninth school year. The majority of our junior high schools will probably find it impossible, for years, to reach the art of geometric demonstration. It is partly a question of mathematical maturity, but more especially one of time schedules. In defense of the earlier introduction of a few logical proofs the argument has been advanced that numerous pupils who leave school at the end of the ninth year should not be deprived of a bowing acquaintance with that unique spirit which has been the glory of demonstrative geometry. A very beautiful conception; but is such a procedure really feasible under existing conditions? Is it pedagogically worth while? The opposing argument has been offered that for similar reasons one might desire to give to these departing boys and girls a few lessons in Greek, or physics, or harmony, or a rapid panoramic view of the world's other great intellectual treasures. After all, there will always be those who turn from the arduous cultivation of the mind to fields of immediate utility.

Conclusion.

The brief review attempted above intends to convey the impression that, on the whole, the tentative syllabus submitted by the National Committee is a sane and constructive piece of work. It is in harmony with the experience and the ideals of progressive teachers, and it can therefore be regarded as a reliable guide to those who wish to assist in the future development of junior high school mathematics.

ROCHESTER, N. Y. WILLIAM BETZ.

DISCUSSION OF PART VI.

A discussion of the Preliminary Report of the National Committee seems to the writer to be necessarily commendatory

in its nature. Comment seems like painting the lily white. The report is a great forward step in our profession.

Part VI, to which I would call attention for the moment, gives additional evidence of the good sense of the document in suggesting several arrangements of the material. This makes the course adaptable to a great variety of schools and conditions.

Personally, I covet the privilege of recommending most heartily Plan A. Its appeal is logical and pedagogical. It is logical intrinsically and in its relation to the purposes of the Junior High School. It is pedagogical in its psychological arrangement of the material.

The child in the seventh school year is mentally able to handle such arithmetic as is needed in practical applications to home and school life. He, furthermore, is beginning to have a care for such problems. He can grapple with simple budget making and percentage relationships and should be encouraged to do so.

Also at this stage in his career he needs training in handwork and the arrangement of his written material. He is interested in form—size and shape. A fortuitous combination of training in these directions finds its expression in Intensive Geometry.

When the pupil has reached the eighth year, he is ready to generalize his views to a certain extent. He can look at abstractions as such to a limited degree. Therefore, he can enlarge the scope of his seventh year problems and classify his facts. He is ready to make and use formulas and equations. He is ready to get the concept of function. He is ready to express relationship graphically. In short, he is mentally capable of and is interested in algebra as a shorthand and a problem solver.

In his ninth school year, the student has acquired a beginning of a logical sense and is entitled to a peep into demonstrational geometry. Obviously, this subject for pupils of this age must be approached very gradually. Moreover, it must have a practical aspect by means of the type of exercises selected. It must also be given with a very limited syllabus. In short, an extensive view of the subject should be given in

this year and should not be the final treatment of the work in the secondary school. The report calls for trigonometry. This undoubtedly implies merely a chapter of Plane Geometry applying the work of similar triangles to the solution of simple trigonometric problems.

Thus, in steps adapted to the interest and maturity of the pupil, the types of secondary school mathematics have been prescribed to the student of the Junior School. Opportunity has been given to find his interest and his ability. The mathematics of the secondary school has been presented in extensive fashion preliminary to the intensive study of these same topics in the Senior High School.

Should the school career of the pupil be terminated at the conclusion of his Junior High School, he has been put in possession of the mathematical tools for practical work. Herein, another purpose of the Junior High School has been fulfilled.

It is interesting to note that in achieving as hinted the purposes of the Junior High School, the subject of mathematics has not been logically maltreated. The sequence of topics, for example, is quite as logical as pedagogical and psychological.

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DISCUSSION OF THE FOUR PRECEDING ARTICLES.

The publications of the National Committee on Mathematical Requirements represent either detailed investigations, or else the composite thought of a very large number of teachers who cooperated with the Committee. The discussion by a member of the Committee can express only the opinion of that individual. With these important facts in mind, it may be worth while to review the preceding articles, and to summarize the points which imply some degree of difference of opinion.

Turning to the first of these articles by Mr. Breslich, we come sharply to a vital issue,—correlation. Mr. Breslich says: "Although correlation is not advocated by the Committee, the second, third and fourth points indicate the possibility, or even the advisability of correlating algebra, geometry, arithmetic and trigonometry," and later, "the report does not

show that the Committee have considered the plan, at least not sufficiently to include it as one of the possible arrangements worked out in part six of the report."

This conclusion is clearly in error. "Plan E" of Part VI consists of the following:

"Plan E."

"First year: Intuitive geometry, simple formulas, elementary principles of statistics; arithmetic (as in Plan A).

Second year: Intuitive geometry; algebra.

Third year: Intuitive geometry; numerical trigonometry; arithmetic."

"By this plan the work of the first two and one half years may be described as general mathematics, while the last half year would be devoted to the special civic and economic features of business practice."

Surely this plan at least demands correlation as one solution. Incidentally, it is to be hoped that the minutes of the National Committee will show that "Plan E" as printed involves an error in proofreading. It would seem that in addition to the contents stated, the second year should include arithmetic also, and that algebra should be added to the third-year list.

Even if "Plan E" were entirely omitted, Mr. Breslich's concern would not be justified. In all the plans proposed, the work of each year is not to be devoted wholly to a single field such as arithmetic or algebra. Keeping in mind that there is no attempt to prescribe the order of presentation of the topics included, there is certainly ample freedom for the ideas and principles of the various fields to be so taught as to reinforce each other.

The real vital issue so far as concerns Mr. Breslich is the fusion of demonstrative geometry with materials chosen from arithmetic, algebra and the like. If the intuitive geometry of the report is taught, the pupil will come to the consideration of demonstrative geometry knowing most of the vocabulary and the symbolism. Moreover, he will have gained conviction of many of the significant geometric relations, and will also have acquired some facility in their use.

Most teachers will probably feel that the only thing worth while (*if any*) that is left is the logical unity, and the method of attack in solving problems dealing with more or less similar materials and situations. Is anything gained in this development by constant interruptions of materials chosen from algebra or other fields? This is the debatable issue on which the National Committee takes no clear definite position. The matter must be settled by scientific experiment, rather than by the assertion of one individual, or that of a group. Possibly the most helpful part of the report on this question is the sentence on page 9: "It is necessary, however, that those propositions which are to be used as the basis of subsequent formal proofs be explicitly listed and their logical significance recognized."

Miss Gugle issues a word of warning "not to over stress such topics as indirect cases of percentage, foreign money, stocks and bonds, and statistics." Since no one possesses reliable information concerning the social values of the topics in arithmetic, one "warning" is as good as another. "Statistics" seem to show that at present (as well as for a generation past) a considerable portion of our citizens have stock and bond investments. In the discussion of the cases of factoring, Miss Gugle seems to express no real disagreement concerning the content. Her statements appear to deal with the method of teaching these cases, a matter which the Committee did not include.

On the unit of demonstrative geometry, Miss Gugle says, among other things, that "care should be taken to choose theorems in which the pupil has something to prove, not self-evident facts and assumptions." It is precisely to make this possible that we read on page 9 of the report: "It should be noted that from the strictly logical point of view the attempt to reduce to a minimum the list of axioms, postulates, or assumptions is not at all necessary, and from a pedagogical point of view such an attempt is very undesirable."

Mr. Betz questions "the obligatory use of the T-square and triangle." Possibly, he is correct, but it would seem that most communities could furnish a T-square, and that usually some boy would be willing to make a simple wooden or cardboard triangle.

We may leave this, and all other matters of minor importance in Mr. Betz's paper, and come to the consideration of his paragraph on demonstrative geometry. His first sentence, "there are a few good American schools, usually private or endowed, that succeed in doing some excellent work in demonstrative geometry in the 9th school year," may be questioned on the basis of fact. It is sufficient to point out that the junior high school series of mathematics texts, which enjoys widest use in *public* schools to-day, contains a unit of demonstrative geometry. Later in the paragraph we read "the opposing argument has been offered that for similar reasons one might desire to give to these departing boys and girls a few lessons in Greek or physics or harmony, or a rapid panoramic view of the world's other great intellectual treasures." While many mathematics teachers (including the writer) are not fully convinced of the wisdom of including demonstrative geometry in the junior high school, the issue certainly demands less fallacious argument. "A few lessons in Greek, or physics or harmony" are wholly false analogies, whereas, if by "a rapid panoramic view of the world's great intellectual treasures" is meant a preliminary consideration of the simple and significant principles of the various fields of human achievement, as, for example, science, art, and the like, the project may be both feasible and desirable. Mr. Wells' recent attempt to give the general reader this panoramic view in "The Outline of History" suggests a task that seems worth doing in all fields, including history, if it should turn out that Mr. Wells has not arrived.

Mr. Walsh, in recommending "Plan A" sees in the work of the National Committee an achievement which it did not consciously attempt. He says, "Its (Plan A) appeal is logical and pedagogical," and in the last paragraph of his paper we read, "the sequence of topics (referring to junior high school mathematics) is quite as logical as pedagogical, and psychological." If all this is true, it must be so by chance, for the Committee did not consciously attempt to secure these high values. It realized that much detailed and exacting work must be done in the class rooms with children by many groups of teachers before the program, suggested as the outcome of

composite opinion, can be realized. It is necessary to emphasize here that the Committee did not attempt to list the order of topics, or even the order in which arithmetic, algebra and geometry should be pursued in any one year of "Plan A," or in any other plan.

It is gratifying to see to what extent differences concerning the junior high school problem are being ironed out. This happy outcome is largely due to such thoughtful attention and constructive criticism as are illustrated by the four preceding papers. The outlook for secondary school mathematics is very promising, and of the various divisions, that of the junior high school is particularly attractive.

THE LINCOLN SCHOOL,
NEW YORK CITY.

RALEIGH SCHORLING.

FIRST LESSONS IN DEMONSTRATIVE GEOMETRY.

It is considered good pedagogy to make the approach to any new subject by means of related subjects of more elementary character. The first few lessons in Geometry need furnish no exception to the above rule. From his work in mensurational arithmetic, the pupil has a fair conception of areas of rectangles, triangles and circles, and volumes of some of the solids. From his experiences in everyday life, he has some idea of the fundamental meaning of the terms: point, straight line, angle, right angle, etc. From Algebra he has acquired the ability to choose symbols wisely and to use them skilfully, and has discovered how valuable they are in shortening operations. All of this fund of information may be used to great advantage by the careful teacher in the introductory lessons of Geometry.

The first class period should be devoted to the acquiring of the more common terms of the new subject. The technical words needed to discuss solids, surfaces, straight and curved lines, angles and angular measurement may well be introduced at the very beginning. A chalk box, the room in which the class works, or any other rectangular solid will furnish sufficient illustration. It is not wise to require too rigid definitions during the first few days. Accuracy of statement and clearness of language will come gradually after the beginner realizes the need of such accuracy. Freedom of discussion is more desirable at this point than rigidity of definitions. By skilful questioning the teacher should be able to gather from the different members of the class some good expressions of ideas that may be used a while at least for definitions.

For the next lesson the student should begin the solution of a large variety of exercises and questions dealing with the new language he has acquired and especially with the numbers of degrees in complementary and supplementary angles, adjacent angles, angles around a point and about a point on one side of a line. The use of Algebra may enter here and the teacher has a splendid opportunity to review simple equations and

give the class a clear idea of many of the terms of Geometry at the same time. This work with a review of the terms already learned may be continued into the third or fourth recitation.

The next thing to be taken up is the work in construction. The tools of Geometry are the straight-edge and compasses and their use is limited to the well known postulates. Early in the course the student should know positively that tools other than these are used for experimental purposes only, as well as the use of these two tools beyond the limits of the postulates. There are many construction exercises the pupil can do at once. He should spend some time comparing lengths of given line-segments, constructing line-segments equal to the sum or difference of given segments, and constructing triangles whose sides are of given lengths or equal to the sides of a given triangle.

The protractor is next to be introduced and considerable experimental work should be done with it. The student should learn to use it in drawing angles of a given number of degrees and in measuring given angles. He should become familiar with several propositions of Geometry by means of its use and in other experimental ways; such as the theorems giving the sum of the angles of a triangle, the sum of the angles of a quadrilateral, the relative values of vertical angles, and the value of each angle of an equilateral triangle. It must be kept before his mind, however, that he has not proved any of these facts. That is, if he has tested five triangles, and finds that the sum of the angles of each is 180 degrees, he has not proved it true of every triangle. The use of the protractor need not be continued beyond the first few lessons. It is not, strictly speaking, a tool of geometry but is a valuable aid in developing a clear conception of the values of angles. A good student of Geometry should be able to estimate within a few degrees the value of any angle drawn on the blackboard. The interest aroused by the use of the tools of Geometry in construction and experiment may serve to awaken his powers of invention. There is in each child's mind an eagerness to find out things for himself. This desire is pretty well obliterated by our methods of feeding him information in huge doses without

giving him the "why" of the information. Let us stop all this in Geometry and give him the satisfaction of deriving his own facts. Truths discovered by the pupil through his own efforts, even though hours of time are required, have an influence that is far-reaching and life-lasting.

Reasonable attention should be given the axioms and a few of the remaining definitions. Also a number of preliminary theorems should be proved informally in class before the formal proofs are begun. Not all the terms that are to be used in Geometry are to be defined during the first few lessons but only those that are immediately necessary. In fact it is very easy to overdo the work with the definitions. If the teacher takes a few minutes each recitation for a review of the meanings of the more important terms used, each pupil will soon have a valuable mathematical vocabulary without the apparent ill effects of too much memorizing.

As soon as the class is ready to begin the proofs of the theorems, it is a wise plan for the teacher to explain the meaning of a proof. Probably many of the failures in Geometry are caused by a misunderstanding of this very thing. Many teachers of this subject would be amazed if they could know what passes in the pupil's mind during the first few lessons of formal proofs. It requires much patience and considerable time for the teacher to get all the members of the class to realize that a completed proof is a chain of facts leading directly from a definite hypothesis to a definite conclusion. Each of these facts must be defended by a reason selected from one of the following geometrical sources: (1) the axioms, (2) the definitions, (3) the hypothesis of the theorems under consideration, and (4) the theorems and corollaries already proved.

There are various ways of accomplishing the desired results mentioned above. One is to make an exhaustive study of one of the earlier proofs that goes directly from the hypothesis to the conclusion in not more than five or six "steps." The class will be interested in studying how one step with its reason leads to the next and that to the next till the conclusion is the final step in the argument.

There are further suggestions which might be made for the younger teacher:

(*a*) Spend a part of a period in asking the members of the class to give the hypothesis and conclusion of several simple theorems.

(*b*) Require that all reasons, for a time at least, be given in full and complete sentences. Careless and slipshod English has no place in a course in Geometry.

(*c*) Compare the solution of a simple linear equation with the proof of a theorem. Notice particularly that each step in the solution may be defended by one or more axioms.

(*d*) Outline the proof of a theorem into four definite parts: (1) the exact statement of the theorem to be proved, (2) the "Given" clause which describes the figure and applies the hypothesis, (3) the "To prove" clause which explains what is to be proved, and (4) the proof.

Before this paper is brought to a close, it is necessary to explain that it has been written with those pupils in mind who take up the study of Geometry as a new subject after a year of elementary Algebra. There is no intention on the part of the authors to argue that that order of studies is the best plan. Because the majority of schools offer the subjects in that order is sufficient reason for the suggestions in the preceding paragraphs. For those schools that offer any form of elementary Geometry in the eighth or ninth year, or that teach any kind of correlated mathematics, these suggestions should of course be modified.

EVANSTON, ILL. M. J. NEWELL,
KENILWORTH, ILL. G. A. HARPER.

EDITORIALS.

With this issue the Mathematics Teacher becomes the official journal of the National Council of Teachers of Mathematics. Its editorial policy will be characterized, it is hoped, by a broad, generous disposition to give approval and publicity to all constructive efforts to improve the teachings of mathematics. In this period of reconstruction, when the whole problem of the selection and organization of the materials in mathematical curricula is so vital, and, to date, so much a matter of a priori thinking, we can ill afford to try to negative the tentative solutions of those whose judgments differ from ours. The Teacher represents no one faction, no one "school of thinking." Its columns shall constitute a forum through which the positive claims of the advocates of any "prescription" of mathemathical material may be discussed. We have a deep conviction that only after clear a priori thinking has been supplemented by long and careful experimentation, can any purported solution of our curriculum problem assert serious claims for wide acceptance and adoption.

Through its various departments (news and notes, reviews, questions and answers, and contributed articles) the Teacher will render an account of the important movements and the best practices in the field of Junior and Senior High School mathematics.

J. R. C.

Prior to 1908 the Association of Teachers of Mathematics in the Middle States and Maryland published an annual bulletin which contained some of the papers presented at its meetings, but the writer felt that there was great need for a periodical issued several times a year and devoted to the interests of the teaching of mathematics and proposed to the Council of this Association that such a periodical be undertaken. The Council agreed to the proposal and in September, 1908, the first number appeared. For twelve and one half years now the duties of

Editor-in-Chief have fallen upon my shoulders, and, perhaps, no one appreciates better than I the shortcomings of the work. Had it not been for the splendid services rendered by my associates the results would have been far less satisfactory. It is hoped that some good has been accomplished and that much more will be in the future under the new management. In order that the magazine may have a wider constituency and serve better the purpose for which it was established, it is with this issue turned over to the National Council.

During these twelve and a half years the magazine has gone to every member of the Middle States and Maryland Association, and for several years now the Association of Teachers of Mathematics in New England has used it as their official organ and during the time it has gone to every member of that Association. The subscription list, independent of the membership of the two Associations, has grown from nothing to about six hundred. Subscribers are to be found in Africa, India, China, Japan, Russia, Finland and in fact in all parts of the world.

W. H. M.

NEWS AND NOTES.

To Dr. W. H. Metzler, more, perhaps, than to any other one person, is the National Council indebted for its promise of success. In the last analysis, the Council will be retarded, or promoted, in the degree to which its journal serves the needs of teachers of mathematics. Dr. Metzler, former editor of the MATHEMATICS TEACHER, saw not only the necessity of the Council having a strong journal, but saw in the Council the possibilities of greatly enlarging the usefulness of the TEACHER. He accordingly helped effect a reorganization of the Editorial Board, by which the TEACHER became the journal of the National Council. Dr. Metzler continues a vital connection with the management of the TEACHER in the capacity of Chairman of the Board of Contributing Editors. This board will determine the general policy of the TEACHER. In this capacity the TEACHER is assured of the wisdom and support of a recognized leader who, almost without assistance, has for twelve years edited the best American journal covering the field of secondary school mathematics.

CHARLES M. AUSTIN, President of the National Council of Teachers of Mathematics, is head of the department of mathematics in the high school at Oak Park, Illinois. He was the moving spirit in the organization of the Chicago City Mathematics Club, as well as of the National Council. Mr. Austin has long been known as a vigorous leader in the movement to vitalize high-school mathematics.

DR. HAROLD O. RUGG, Vice-President of the Council, is Educational Psychologist in The Lincoln School of Teachers College, New York City, and Editor-in-Chief of the *Journal of Educational Psychology*. Dr. Rugg has been associated with John R. Clark in a series of investigations in the reorganization movement in secondary mathematics.

JOHN A. FOBERG is probably our best known high school teacher of mathematics. He is Secretary-Treasurer of the

National Council; Business Manager of the MATHEMATICS TEACHER and Vice-Chairman of the National Committee on Mathematical Requirements. The state of Pennsylvania has induced him to leave the Crane Technical High School, Chicago, to become State Director of Mathematics. Mr. Foberg, the State of Pennsylvania and the cause of mathematics are to be congratulated.

DR. JONATHAN RORER, of the Central High School, Philadelphia, is ably representing the Council, as a member of the Executive Committee, in his section. Dr. Rorer gives courses in the teaching of Junior High School Mathematics in the Johns Hopkins University. Many Baltimore teachers are enrolled in the courses.

MARIE GUGLE, assistant superintendent of schools in Columbus, Ohio, represents the Council both as a member of the Executive Committee and as an editor of the TEACHER. Miss Gugle has been active in the Central Association of Science and Mathematics Teachers.

W. D. REEVE, of the University of Minnesota High School, is irresistible in his enthusiastic campaign for the Council. Through his efforts more than one hundred and fifty teachers in Minnesota have joined the Council.

W. A. AUSTIN, of the Fresno, California High School, represents the Council in the far west. He has been a contributor to various journals.

MR. W. E. BECK, of Iowa City, Iowa, has been very active in the Central Association of Science and Mathematics Teachers. He has presented the plans of the Council to many teachers in the Middle West.

ANY enumeration of those who have been active in the organization of the National Council would be incomplete if the names of M. J. Newell of Evanston, Ill., E. R. Smith of Baltimore, D. W. Werremeyer of Cleveland, Raleigh Schorling of the Lincoln School, W. C. Eells of Walla Walla, Washington, D. E. Smith of Teachers College, W. W. Gorsline of Chicago, Alfred Davis of St. Louis, W. E. Breckenridge of New York, and William Betz of Rochester, were not included.

"THE annual meeting of the Association of Teachers of Mathematics of the Middle States and Maryland was held at the Johns Hopkins University, Baltimore, Maryland, on November 27, with one hundred and ten persons in attendance. Dr. Metzler, Editor of the MATHEMATICS TEACHER from its beginning until its transfer to the National Council, explained that members of the Association would receive the January and February numbers of the TEACHER because of their membership in the Association, but that members would have to arrange with the National Council for the March number and subsequent issues. At the conclusion of his remarks Dr. Metzler was tendered a vote of thanks by the Association for his labors on the TEACHER. In the opening remarks of President Breckenridge, in the remarks by Dr. Metzler, and throughout the meeting the sentiment was continually expressed that the Association should accord the Council enthusiastic support because the Council, representing the whole country, may be able to secure action on reforms desired by the Association.

"Dean Hawkes, of Columbia, discussed the report* of the National Committee on new requirements for admission to college. The speaker gave a very interesting discussion of the spirit of the report. Two-minute discussions of Dr. Hawkes' remarks were made by Dr. Rorer, Mr. H. F. Hart, Mr. Raleigh Schorling, and Miss Baker.

"The remainder of the program of the morning was given over to a discussion of the Report (Circular No. 6) of the National Committee on Junior High School Mathematics. Mr. E. R. Smith, Headmaster of the Park School, Baltimore, opened the discussion, speaking on the literature and general considerations on which the report is built. Miss Annabel White, of the Western High School, Baltimore, discussed experiences in Baltimore with the type of mathematics advocated in the report in the junior grades. Dr. J. T. Rorer, of the William Penn High School, Philadelphia, followed with the discussion of the uselessness of too early specialization in the junior grades and the distinguishing characteristics of present

* It is expected that members of the Association will receive this report when it is printed and distributed by the Bureau of Printing at Washington.

texts in the junior high school field. Dr. Harry English, supervisor of mathematics in Washington, closed the discussion with a comparison of the likenesses and differences of the reports of the National Committee on Junior High School Mathematics and that on first courses for high schools (Circulars 5 and 6).

"Mr. Raleigh Schorling, of the Lincoln School, New York, read the paper of the afternoon on "New Experimental Courses in Mathematics." This paper was a fitting climax to an interesting day. He discussed the curricula in mathematics in use in such schools as the Cass Technical School, Detroit, the school connected with the University of Missouri, the ideas contained in the Rochester, N. Y., curriculum, and others. There was much regret that time did not permit of his speaking at any length of the course in the Lincoln School." (Contributed by H. F. Hart, Montclair, N. J.)

THEODORE LINDQUIST, one of our cooperating editors, published in the October, 1920, issue of *School and Society* an important article on the "Application of Business Principles to Junior High School Mathematics." Convincing argument is advanced for deferring until later in the course, than has customarily been given, the treatment of certain phases of business practice.

"WHY Study Mathematics?" is the title of an unusually cleverly written article published by Professor Arnold Dresden of the University of Wisconsin in the October 30, 1920, issue of *School and Society*. Through conversational style the author tells the reader what the mathematician means by *functionality*.

THERE is increasing emphasis on the *problem method* of teaching. Readers who are interested in the literature of this method will welcome Professor Parker's series of articles on "Problem Solving or Practice in Thinking," which appeared in the 1920 September, October and November issues of the *Elementary School Journal*.

THE Chicago teachers of high-school mathematics have been working on an improved curriculum for the ninth grade. Mr. Olice Winter of the Harrison Technical High School can supply information relative to this investigation.

"THE Mathematics Section of the Indiana State Teachers' Association met in Indianapolis, October 21. The program consisted of: Report of Committe to Cooperate with the National Committee on Mathematical Requirements. "High School and College Mathematics," Thomas E. Mason, Purdue University. "Tests and Examinations—Their Uses and Abuses," J. A. Reising, Fort Wayne High School. "Mathematics—Theoretical or Practical," Jesse E. Adams, County Supt. of Knox County." (Contributed by F. H. Croninger, Fort Wayne, Ind.)

"THE program of the first meeting of the Chicago Mathematics Club on the evening of October 14, included: (1) Claims of the National Council, Mr. C. M. Austin, President of the National Council. (2) An Introduction to a Number System, Prof. E. H. Moore, Chicago University. On November 12 the program was: (1) Outcome of the questionnaire among the Chicago teachers on "What Shall Constitute the Material of the First Year in Mathematics?", Mr. Olice Winter, Harrison Technical School. (2) Discussion of the report of the National Committee on Junior High School Mathematics, Mr. J. A. Foberg." (Supplied by M. J. Newell, Evanston, Ill.)

J. C. BROWN, who is contributing "Geometry in the Junior High School" in the February issue, is the author of a report on "Comparative Curricula in Mathematics." This report, published by the Bureau of Education, is one of the most helpful sources of suggestion for Junior High School Mathematical Curricula.

JOHN CROWELL, of the Swift Continuation School, Chicago, made an inquiry of the department heads in the establishment of Swift and Company concerning the specific kinds of mathematical skill and knowledge which they thought their employes ought to possess. This investigation was reported to the Chicago Mathematics Club at the April meeting.

MORE than one hundred teachers attended the December meeting of the Cleveland City Mathematics Club.

QUESTIONS AND ANSWERS.*

Q. 1. I note that the report of the National Committee attaches considerable importance to certain elementary topics in statistics. Where can a teacher who has not had the advantage of a course in this subject obtain information regarding the fundamental principles of the science of statistics?

J. H. A.

Q. 2. Are any accredited high schools of the country giving courses in applied or practical mathematics as a substitute for, and not in addition to, the traditional algebra and geometry? If so, are such courses acceptable for entrance credit at Universities of good standing?

H. D.

Q. 3. I note that the National Committee has recommended that a rather large amount of intuitive geometry, considerable algebra, and some numerical trigonometry, be given before demonstrative geometry. Assuming that this is done, will there be any justification for correlation in the demonstrative geometry?

H. B. K.

Q. 4. I have been unable to distinguish clearly between *correlated*, *fused*, and *general*, mathematics. Will you explain the meaning of each of these?

W. A. A.

* This department of the TEACHER is conducted by the Associate Editor, Eugene R. Smith, Headmaster of the Park School, Baltimore, Md. All correspondence should be addressed to Mr. Smith. Readers are invited to submit questions or answers in the field of their special interests. The editor in charge will refer questions to persons specially qualified to answer them. Some, or all, of the following questions will be answered in the February issue.

NEW BOOKS.

The Mannheim and Polyphase Slide Rule. By WILLIAM E. BRECKENRIDGE. New York City: Keuffel and Esser Co. Pp. 80.

This is the first textbook on the slide rule designed for use in schools from the eighth grade through college. It is self-teaching by means of numerous cuts and detailed instructions. A teacher will find that he can easily teach himself and his class the use of the slide rule by the aid of this book. Numerous examples and problems with answers furnish ample material for drill. The book is designed for the use of teachers of Trigonometry, Algebra, Arithmetic, and Practical Mathematics in colleges, high schools, and junior high schools.

Illustrated Mathematical Talks by Pupils of the Lincoln School. Pp. 48. Distributed by the school, 646 Park Ave., New York City.

Concerning this booklet, we quote in part from an editorial in the December 2, 1920, issue of *Journal of Education:*

"A REMARKABLE DEMONSTRATION.

"No more interesting booklet has come to our desk than 'Illustrated Mathematical Talks by Pupils of the Lincoln School, New York.'

"We have previously called attention to the fact that Otis W. Caldwell, principal of the school, has occasional parents' meetings in which the pupils present in their own way specific illustrations of real school work. The latest of these demonstrations is reported in a remarkable booklet.

"It is an honest report of what the pupils did and how they did it. It is impossible for us to give any adequate suggestion even of the value of this booklet. For instance, there is exquisite color work in which the original designs in color are reproduced.

"On one page are six beautiful colored reproductions of designs by six pupils in the Fine Arts class. There are elaborate articles, 'Fundamental Principles in the Construction of Graphs,' by a lad of thirteen; 'Discussion of Graphs in Common Use,' by a lad of fourteen; 'Use of Graphs in School Subjects,' by a lad of twelve; same subject extended, by a girl of twelve; 'Graphic Solution of Interest Problems,' by a lad of fourteen; 'Signed Numbers,' by a girl of fourteen; 'The Origin of Mathematics,' by a lad of twelve; 'Early Shelters,' by a lad of eleven; 'Weaving,' by a girl of eleven; 'Geometry and Nature,' by a girl of twelve; 'Geometry in Everyday Life,' by a lad of twelve; 'Surveying,' by a lad of fourteen; 'Logarithms: The Slide Rule,' by a girl of fifteen; 'Trigonometric Ratios,' by a girl of fourteen. There are several other similar demonstrations.

"Whatever of super-merit there is in this demonstration would be possible in any school with equally efficient teachers, with the same freedom for initiative and adventure on the part of teachers and children."

SECOND ANNUAL MEETING NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS.

ATLANTIC CITY, NEW JERSEY.

THURSDAY, MARCH 3, 1921.

Afternoon Session—2 to 5 P.M.

Journals of Europe for Teachers of Secondary Mathematics, Dr. David Eugene Smith, Teachers College.

Plans and Policy of THE MATHEMATICS TEACHER, J. R. Clark, Editor, The Lincoln School, New York City.

Round Table Discussion. Topics:

1. How can the National Council best serve the cause of Mathematics?
2. How can THE MATHEMATICS TEACHER be of the Greatest Service to the Teachers of Mathematics?

C. B. Walsh, Friends School, Philadelphia.
W. E. Breckenridge, Stuyvesant High School, New York.
E. R. Hedrick, University of Missouri, Columbia, Mo.
Miss Litia Odell, North Side High School, Denver, Col.
W. W. Jones, North Central High School, Spokane, Wash.

Evening Session—7 to 10.

Illustrated Lecture—Geometry and Appreciation in Nature and Art, W. E. Betz and Raleigh Schorling, The Lincoln School, New York City.

The Work of the National Committee on Mathematical Requirements, Dr. J. W. Young, Dartmouth College.

Discussion:

C. L. Thiele, Supervisor of Arithmetic, Detroit.
Wm. H. McAndrew, Associate Superintendent, New York.
L. W. Colwell, Principal Cleveland School, Chicago.
Miss Mary Faloney, Lattimer Junior High School, Pittsburgh.
E. H. Taylor, Eastern Illinois Normal, Charleston, Ill.
J. Calvin Funk, Polytechnic High School, Mill Valley, California.